# Spontaneous symmetry breaking and periodic structure in a multilane system

Rui Jiang,[1] Mao-Bin Hu,[1] Bin Jia,[2] Ruili Wang,[3] and Qing-Song Wu[1]
[1]*School of Engineering Science, University of Science and Technology of China, Hefei 230026, China*
[2]*School of Traffic and Transportation, Institute of System Science, Beijing Jiaotong University, Beijing 100044, China*
[3]*Institute of Information Sciences and Technology, Massey University, New Zealand*
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We study a multilane totally asymmetric simple exclusion process (TASEP) with narrow entrances under parallel update. The narrow entrances are modeled in this way: the entry of a particle is not allowed if the exit site of the previous lane is occupied. It is shown the results depend on the number of lanes, $n$. If $n$ is an even number, the results are essentially the same as $n=2$: two symmetry-breaking phases—i.e., a high-density-low-density (HD-LD) phase and an asymmetric low-density-low-density phase—are identified. In contrast, if $n$ is an odd number, a periodic structure is observed and the period is found to be proportional to the lane number $n$ and system size $L$. It is also found when the injection rate $\alpha=1$ that the seesaw phase observed in the case of $n=2$ disappears and the HD-LD phase or symmetric LD phase appears. Some mean-field calculations are also presented. We show that it is not possible to have high densities in two successive lanes and also not possible to have high density in one lane and low densities in all other lanes. For odd $n$, we have obtained the period $T$ and it is in good agreement with simulations for a small removal rate $\beta$, but deviates from simulation results for large $\beta$ because correlation is neglected in the mean-field approximation.

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## I. INTRODUCTION

Spontaneous symmetry breaking is an interesting nonequilibrium behavior observed in driven diffusive systems. The first model that exhibits spontaneous symmetry breaking is known as the "bridge model" [1]. In the model, two species of particles move in opposite directions. It was shown that two phases with broken symmetry—i.e., a high-density-low-density phase and an asymmetric low-density-low-density phase—could exist, despite the update rules being symmetric with respect to the two species. While it is argued by Monte Carlo (MC) simulations that asymmetric low-density-low-density phases do not exist in the thermodynamic limit [2–8], mean-field analysis shows it could exist in a very small region.

Following the seminal work of the bridge model, some new results have been reported. Willmann *et al.* [5] investigated the bridge model with parallel sublattice update, and it is shown that the symmetric breaking is due to an amplification mechanism of fluctuations. Popkov and Schütz argued that spontaneous symmetry breaking requires placing effective impurities at the boundaries and therefore does not occur with partial-differential-equation-friendly boundaries [4]. Levine and Willmann found spontaneous symmetry breaking could disappear if the particle attachment and detachment effect is considered [9]. Popkov and Peschel studied a system with two parallel channels. There is no exchange of particles, but the hopping rates in one chain depend on the local configuration in the other one. It is shown that symmetry-breaking phenomena could also appear [8,10]. Recently, Pronina and Kolomeisky [11] and Jiang *et al.* [12] have studied the spontaneous symmetric breaking in a two-lane asymmetric exclusion process (ASEP) system with narrow entrances. It is shown that there are two phases exhibiting spontaneous symmetric breaking as in the bridge model. Note that the model of Pronina and Kolomeisky [11] and Jiang *et al.* [12] could be viewed as a special model with two species of particles where interactions between the two species occur only at both ends.

This paper extends the two-lane system investigated in [11,12] to an $n$-lane ($n>2$) ASEP system with narrow entrances. It is shown the results depend on the value of $n$: if $n$ is an even number, the results are essentially the same as $n=2$; in contrast, if $n$ is an odd number, a periodic structure is observed.

The paper is organized as follows. In Sec. II we give a brief description of the model. In Sec. III, we present the results of MC simulations and some mean-field calculations. We give our conclusions in Sec. IV.

## II. MODEL

The sketch of an $n$-lane model with narrow entrances is shown in Fig. 1. The system consists of $n$ one-dimensional $L$ lattices with particles moving along the direction as shown by the arrows. Hopping between the lanes is not allowed. In the bulk, a particle hops to the next site with probability $q$ provided the target site is empty. At the exit site, a particle is removed with probability $\beta$. At the entrance site of lane $l$ ($l>1$), a particle is injected with probability $\alpha$ provided this site and exit site of lane $l-1$ are empty. At the entrance site

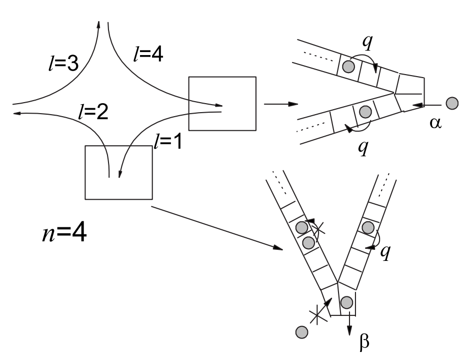


FIG. 1. Sketch of a four-lane model with narrow entrances.

 

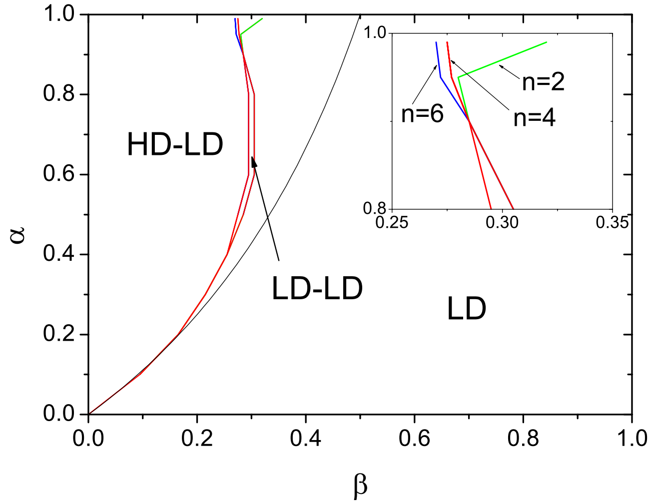


FIG. 2. (Color online) Phase diagram corresponding to even $n$. The inset shows the details near $\alpha=1$. The black line is $\alpha=\frac{\beta}{1-\beta}$ from mean-field calculations. The system size is $L=1000$ in the simulations.

of lane 1, a particle is injected with probability $\alpha$ provided this site and the exit site of lane $n$ are empty. In this paper, parallel update rules are adopted and we focus on the deterministic case $q=1$.

## III. RESULTS

### A. Simulation results

In this subsection, the Monte Carlo simulation results are presented and discussed. Some mean-field calculations are presented in the next subsection. In the simulations, the system is initially empty and a transient time of $10^8$ time steps is discarded. We gather data for $4\times 10^8$ time steps unless otherwise mentioned. We mainly study system size $L=1000$. We investigate the particle density histograms $P_L(\rho_1,\rho_2)$, where $\rho_1$ and $\rho_2$ are instantaneous densities of particles in two successive lanes chosen arbitrarily.

#### *1. Even n*

Figure 2 shows the phase diagram in the space $(\alpha,\beta)$ with even $n$. The phase diagram is classified into three phases: symmetric low-density (LD) phase, asymmetric low-density-low density (LD-LD) phase, and asymmetric high-density-low-density (HD-LD) phase. The typical particle density histograms in the three phases are shown in Fig. 3. As reported before, Monte Carlo simulations show that the LD-LD phase will gradually disappear with an increase of system size.

The phase diagram is independent of $n$ except at large values of $\alpha$. At large $\alpha$, the boundary between the LD phase and HD-LD phase moves slightly to the left with an increase of $n$. We believe the difference is due to a finite-size effect because it becomes smaller and smaller with an increase of system size.

A qualitative change happens at $\alpha=1$. It has been shown in Ref. [12] that a seesaw phase appears when $\alpha=1$ and $n=2$ [see Fig. 4(a)], in which the global maximum could be achieved along the line $\rho_1+\rho_2=1$. This is because when $\alpha=1$, usually a particle will be inserted into one lane after a particle is removed from the other lane. Therefore, the number of particles in two lanes remains unchanged.

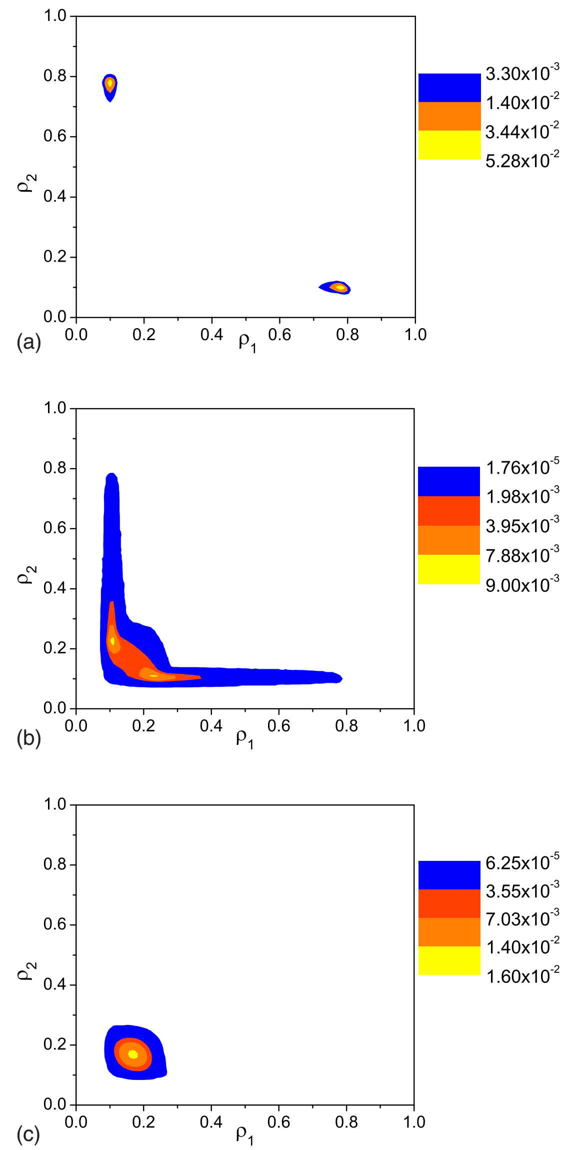


FIG. 3. (Color online) Density histograms at $\alpha=0.5$, $n=4$. (a) $\beta=0.26$, HD-LD phase; (b) $\beta=0.28$, LD-LD phase; (c) $\beta=0.3$, symmetric LD phase.

Nevertheless, when $n\geqslant 4$, the seesaw phase disappears and the HD-LD phase (at $\beta<\beta_c\approx 0.88$) or symmetric LD phase (at $\beta>\beta_c$) is observed at $\alpha=1$ [see, e.g., Figs. 4(b) and 4(c)]. This is because the entry of a particle into, say, lane 1 is determined by the exit site of lane $n$ instead of lane 2.

To demonstrate spontaneous symmetry breaking, we study the flipping times between the two states of the broken-symmetry phase in a finite-sized system. Figure 5(a) shows the time evolution of the density difference between lanes 1 and 2 in a HD-LD phase. Flips between the two symmetry-related states are clearly seen.

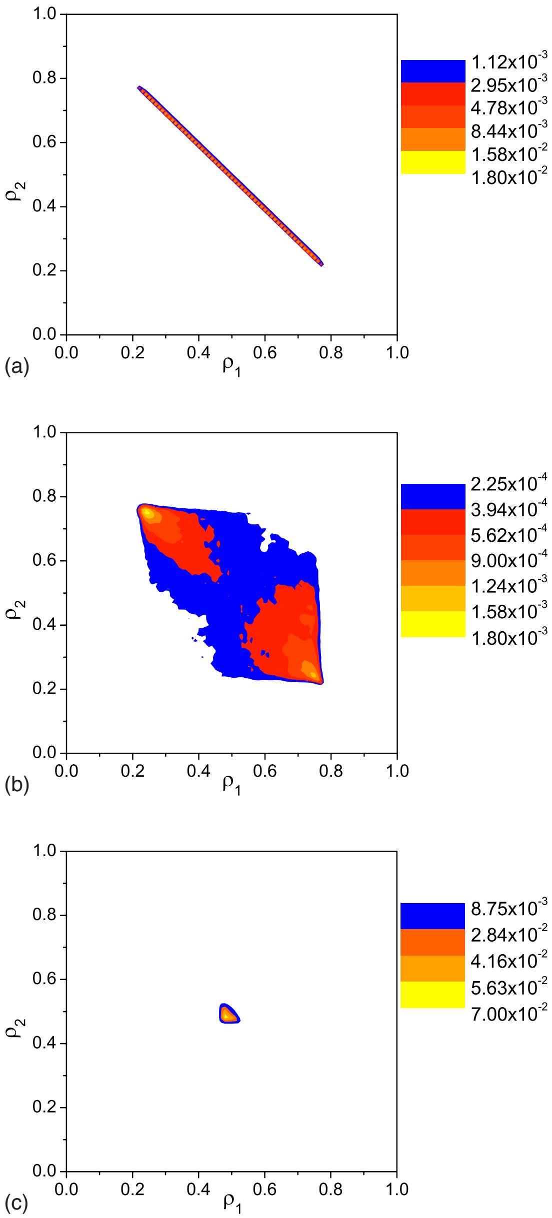


FIG. 4. (Color online) Density histograms at $\alpha=1$. (a) $n=2$, $\beta=0.3$, seesaw phase; (b) $n=4$, $\beta=0.3$, HD-LD phase; (c) $n=4$, $\beta=0.9$, symmetric LD phase.

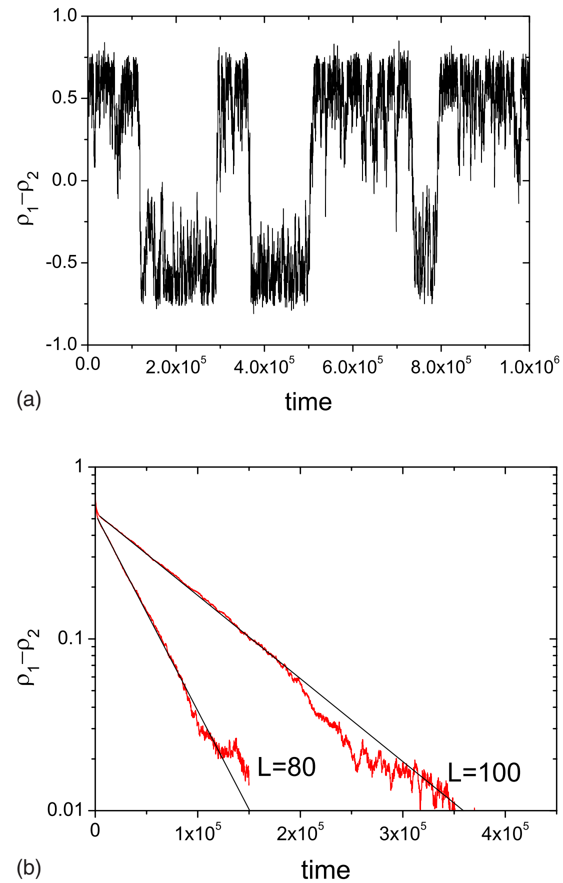


FIG. 5. (Color online) (a) Time evolution of the density difference between lanes 1 and 2 in a HD-LD phase. System size $L=100$. (b) Decay of averaged density difference between lanes 1 and 2 (over $10^4$ runs). The solid line is a guide for the eye. Parameter values: $\alpha=0.5$, $\beta=0.26$, $n=4$.

To evaluate the characteristic flipping time scale $\tau$, we averaged the density difference over many runs, starting from the configuration that all sites on lanes 1,3,5,… are occupied and all sites on lanes 2,4,6,… are empty. This average decays as $\exp[-t/\tau]$ and thus yields $\tau$ [see, e.g., Fig. 5(b)]. Figure 6 shows the time scale versus $L$ for $n=2$, 4, and 6. It can be seen that $\tau$ grows exponentially with system size $L$, which indicates that spontaneous symmetry breaking does exist.

Suppose the system is in a state that all lanes with an even number are in high densities and all lanes with an odd number are in low densities; we study the flipping process for $n \geqslant 4$. It is found, first, that all lanes with an even number synchronically transit from high densities to low densities. Then all lanes with an odd number transit from low densities to high densities [Fig. 7(a)]. In this way, the system transits to the other state. The flipping process will not happen even if one lane with an even number does not transit to low density. In this case, high densities will soon be recovered in all lanes with an even number [Fig. 7(b)].

It is clear that the probability that all lanes with an even number transit synchronically from high densities to low

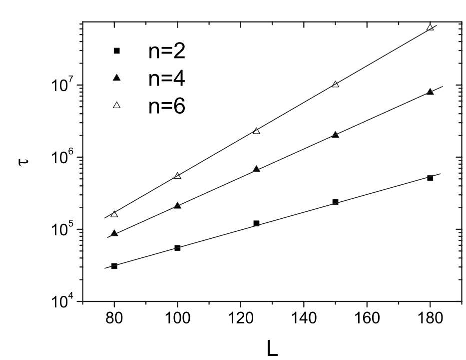


FIG. 6. Characteristic flipping time scale $\tau$ versus system size $L$. The solid lines are a guide for the eye. Parameters values: $\alpha=0.5$, $\beta=0.26$.

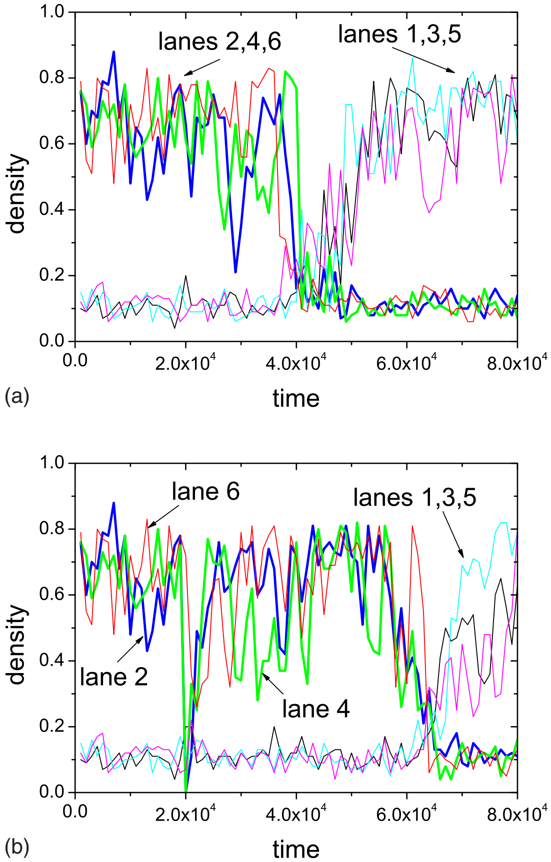


FIG. 7. (Color online) (a) Flipping process in a HD-LD phase. (b) Same as in (a) before $t=2\times10^4$. Then we remove all particles in lanes 2 and 4 at $t=2\times10^4$. One can see high densities are soon recovered in lanes 2 and 4. Parameters values: $\alpha=0.5$, $\beta=0.26$, $n=6$, $L=100$.

densities decreases with an increase of $n$. As a result, $\tau$ increases with an increase of $n$. Our simulations show when $n$ and $L$ are not large, $\tau \propto \exp(bn^\gamma)$. Here $\gamma$ and $b$ increase with an increase of $L$ (Fig. 8). Nevertheless, presently we are not sure the results could be generalized to large values of $n$ and $L$ or not, which needs further investigations [16].

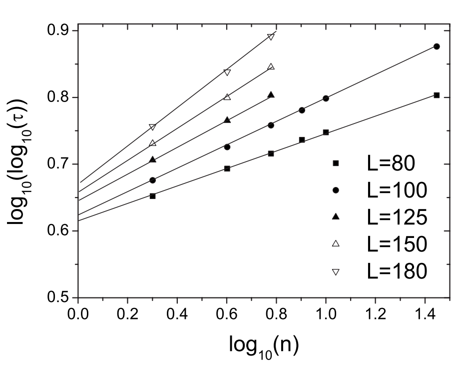


FIG. 8. Characteristic flipping time scale $\tau$ versus lane number $n$. The solid lines are a guide for the eye. Parameters values: $\alpha=0.5$, $\beta=0.26$.

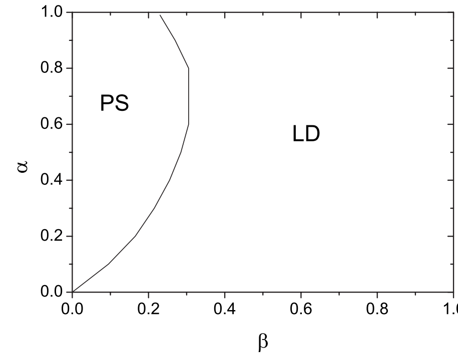


FIG. 9. Phase diagram corresponding to odd $n$. The phase diagram is essentially independent of $n$.

#### 2. Odd n

Next we consider the case of odd $n$. Figure 9 shows the phase diagram in the space $(\alpha,\beta)$ with odd $n$. The phase diagram is classified into two phases: a phase with periodic structure (PS) and a LD phase. Typical particle density histograms of the two phases are shown in Fig. 10. The phase diagram is essentially independent of $n$.

Figure 11(a) shows the evolution of the densities in three lanes for the PS phase at $\alpha=0.8$, $\beta=0.1$, $L=1000$, and $n=3$. We concentrate on densities in lanes 1 and 2 within one period [Fig. 12(a)]. When $T_1<t<T_2$, lane 1 is in high density and the density in lane 2 is decreasing. When $T_2<t<T_3$, low density is maintained in lane 2 and the density is decreasing in lane 1. When $T_3<t<T_4$, low density is maintained in lane 1 and the density is increasing in lane 2. When $T_4<t<T_5$, high density is maintained in lane 2 and low density is maintained in lane 1. Finally, when $T_5<t<T_6$, the density is increasing in lane 1 and decreasing in lane 2. This periodic structure is also manifested in Fig. 10(a) where the arrow shows the evolution process.

When $\alpha$ is fixed, the period decreases with an increase of $\beta$. At the same time, the maximum value of the density decreases [Figs. 11(b) and 11(c)] and the PS structure shown in the density histogram shrinks [Figs. 10(b) and 10(c)]. We note that when $\beta$ is large [e.g., $\beta=0.18$ in Fig. 11(c)], the maximum density that can be reached in each lane in each period becomes inhomogeneous in the simulations. We also would like to point out the period $T$ depends on system size: it increases with system size [cf. Figs. 11(a) and 13; see also Fig. 19 and mean-field analysis in next subsection].

When $\beta$ is fixed, the period $T$ increases (decreases) with an increase (decrease) of $\alpha$ (Fig. 14). We can also see that the periodic structure becomes a little inhomogeneous for large $\alpha$ [Fig. 14(b)]. When $\alpha=1$, the periodic structure could not be reproduced. Instead, a HD-LD phase (at $\beta<\beta_c\approx0.88$) or symmetric LD phase (at $\beta>\beta_c$) is observed (Fig. 15).

We investigate the period $T$ of the PS structure. One can see from Figs. 11(a) and 16 that $T=nT_d$ as shown by the dashed lines. Here $T_d$ is the time for a lane to transit from high density to low density.

Next, we concentrate on densities in lanes 1 and 2 within one period in the case of $n=5$ and $n=7$ [Figs. 12(b) and 12(c)]. As in the case of $n=3$, lane 1 is in high density and

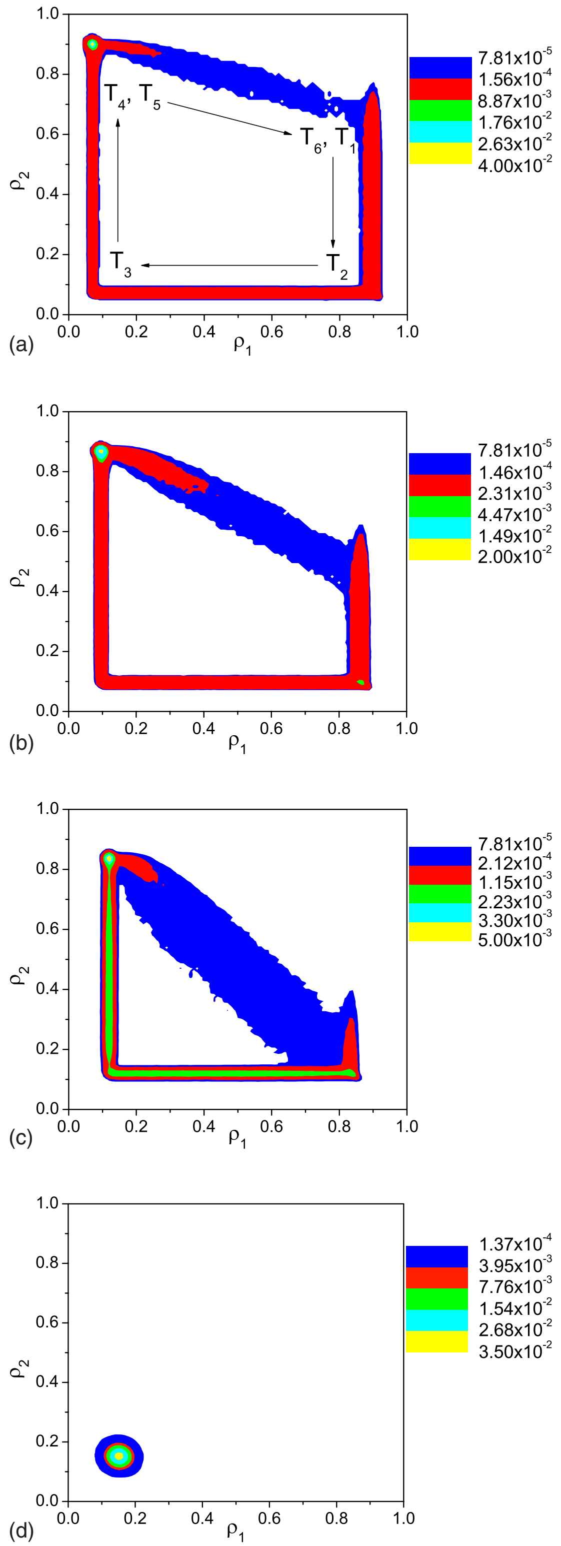


FIG. 10. (Color online) Density histograms at $n=3$. (a) $\alpha=0.8$, $\beta=0.1$; (b) $\alpha=0.8$, $\beta=0.14$; (c) $\alpha=0.8$, $\beta=0.18$; (d) $\alpha=0.4$, $\beta=0.3$.

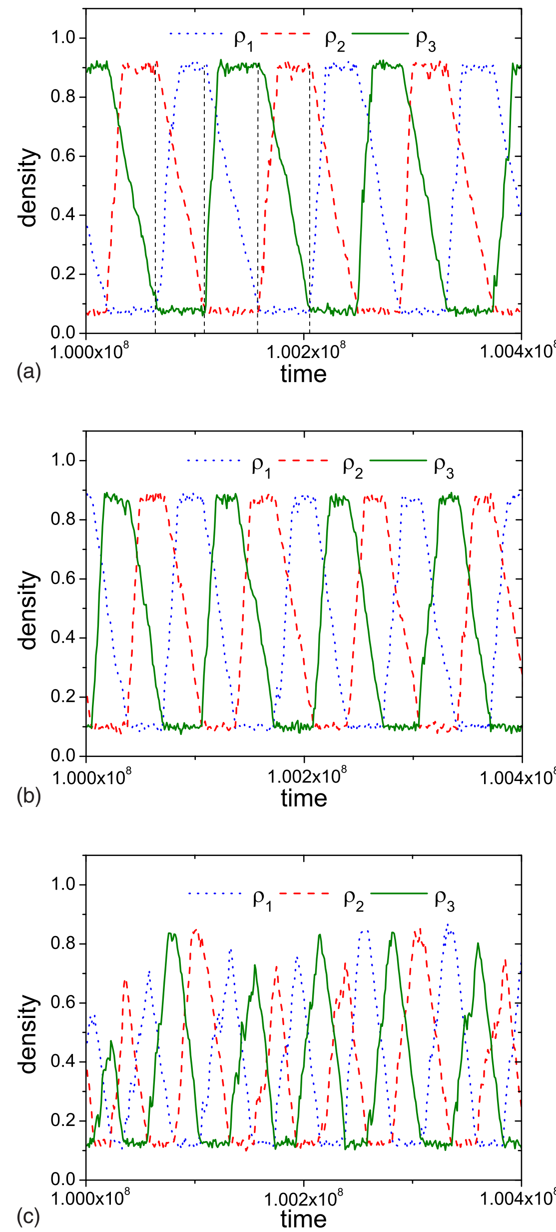


FIG. 11. (Color online) Evolution of the densities in three lanes for the PS phase. (a) $\alpha=0.8$, $\beta=0.1$; (b) $\alpha=0.8$, $\beta=0.14$; (c) $\alpha=0.8$, $\beta=0.18$.

the density in lane 2 is decreasing when $T_1<t<T_2$. But there appears one more state in which a low density is maintained in lane 2 and a high density is maintained in lane 1 ($T_2<t<T_3$). Then the densities evolve similarly to that corresponding to $n=3$. From numerical simulations, it can be easily found that $T_3-T_2=\frac{n-3}{2}T_d$ and $T_6-T_5=\frac{n-1}{2}T_d-T_i$ based on Figs. 12(b) and 12(c) and Fig. 16, with $T_i$ the time for a lane to transit from low density to high density.

Finally, we study the current of the system in the PS phase. We consider the case of $n=3$, and similar results can be obtained for $n>3$. We denote the currents observed at exit sites in lanes 1,2,3 as $q_1,q_2,q_3$ and the currents observed at the entrance sites in lanes 1,2,3 as $q_4,q_5,q_6$. Figure 13 shows the evolution of $q_1$ and $q_4$. When lane 1 is in high density, $q_1=q_4=C_1$ and $C_1$ can be calculated by $\frac{\beta}{\beta+1}$. When lane 1 is in low density, $q_1=q_4=C_2$. $C_2$ can be calculated by Eq. (12) shown below because lane 3 is transiting from high density to low density and the exit site of lane 3 is in high

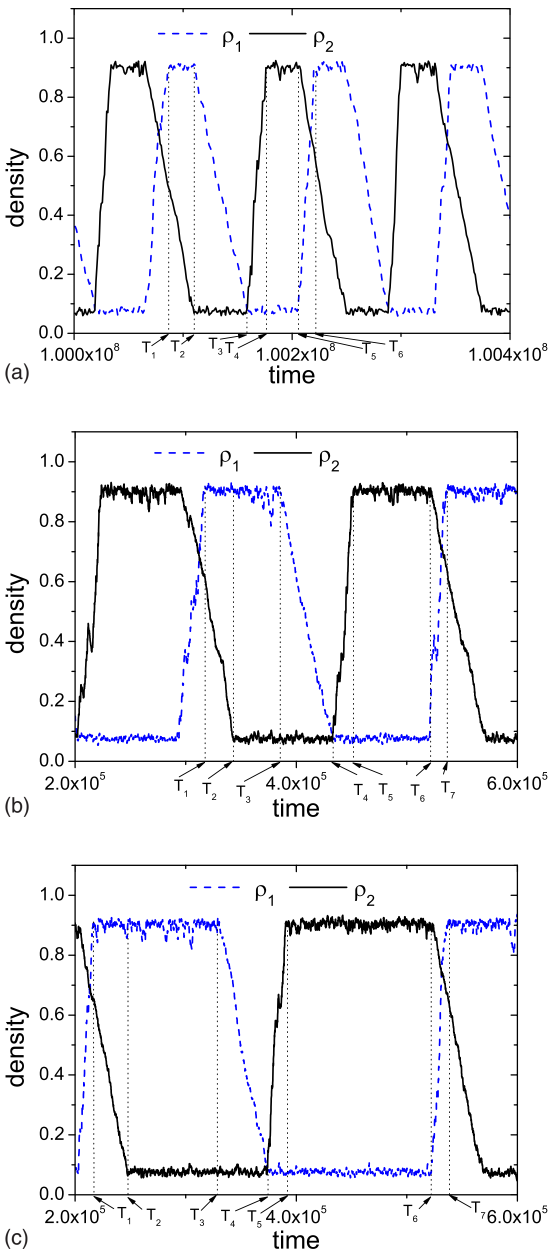


FIG. 12. (Color online) Evolution of the densities in lanes 1 and 2 within one period. Parameters values: $\alpha=0.8$, $\beta=0.1$. (a) $n=3$, (b) $n=5$, (c) $n=7$.

density. When lane 1 is transiting from high density to low density, $q_1=C_1$ and $q_4=C_2$. When lane 1 is transiting from low density to high density, $q_1=C_1$ and $q_4=C_3$. $C_3$ can be calculated by Eq. (15) shown below because lane 3 is in low density. Figure 17 compares the simulation results of $C_1$, $C_2$, and $C_3$ with analytical results. It can be seen that the simulation results of $C_1$ are in good agreement with analytical results. The simulation results of $C_2$ are slightly larger than the analytical results, and the simulation results of $C_3$ are much smaller than the analytical results. This is due to the correlation being neglected in the mean-field approximation. Figure 18 shows the time evolution of $q_1+q_2+q_3$ and $q_4+q_5+q_6$. The former is a constant $2C_1+C_2$; the latter is fluctuating with period $T_d$ between $C_3+2C_2$ and $C_1+2C_2$.

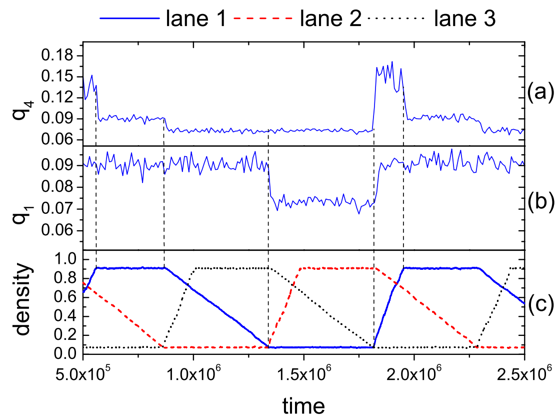


FIG. 13. (Color online) Evolution of (a) $q_4$ and (b) $q_1$ in lane 1. (c) Evolution of the densities in three lanes for the PS phase. Parameters values: $\alpha=0.8$, $\beta=0.1$, $L=10000$.

### B. Mean-field analysis

In this subsection, a brief mean-field analysis is presented and compared with Monte Carlo simulation results. Let us briefly recall the results of the ASEP on a single lane with open boundaries. When the entrance probability is larger than removal probability (i.e., $\alpha>\beta$), the system is in a high-density phase and the bulk density is $\rho=\frac{1}{\beta+1}$; the current is $J=\frac{\beta}{\beta+1}$. When $\alpha<\beta$, the system is in a low-density phase and the bulk density is $\rho=\frac{\alpha}{\alpha+1}$; the current is $J=\frac{\alpha}{\alpha+1}$. The

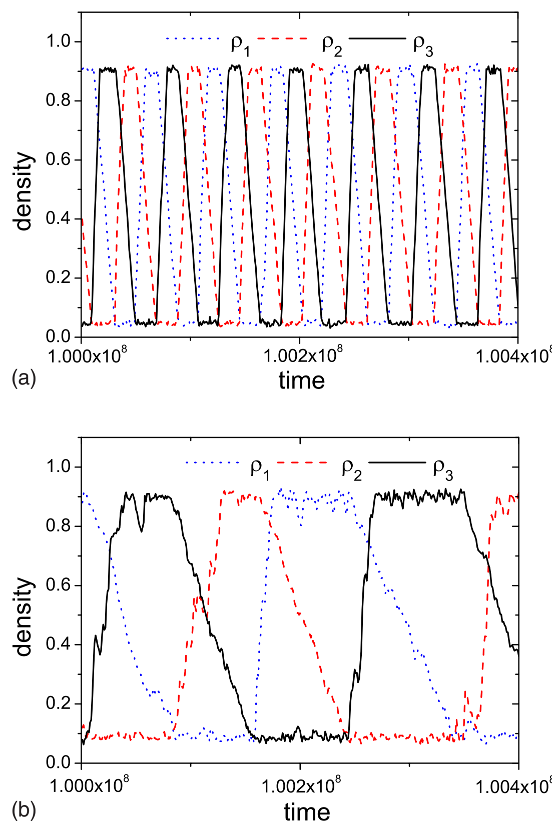


FIG. 14. (Color online) Evolution of the densities in three lanes for the PS phase. (a) $\alpha=0.5$, $\beta=0.1$; (b) $\alpha=0.9$, $\beta=0.1$.

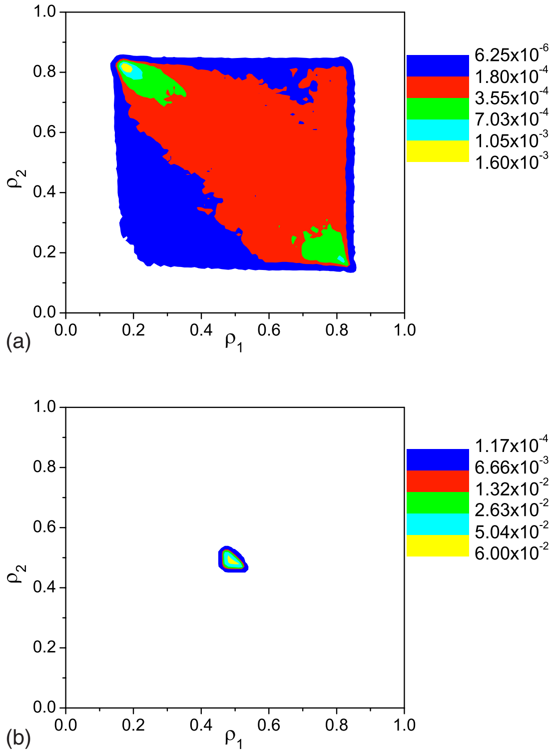


FIG. 15. (Color online) Density histograms at $\alpha$=1, $n$=3. (a) $\beta$=0.2, HD-LD phase; (b) $\beta$=0.9, symmetric LD phase.

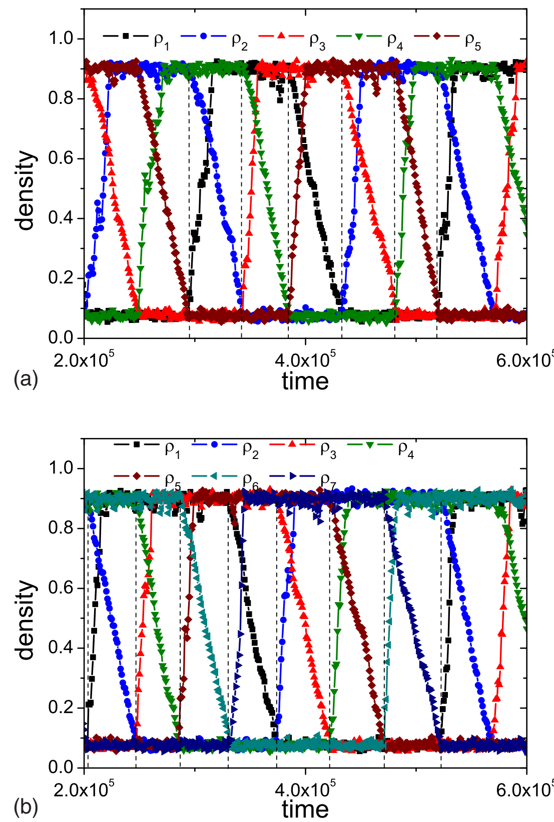


FIG. 16. (Color online) Evolution of the densities in all lanes. Parameters values: $\alpha$=0.8, $\beta$=0.1. (a) $n$=5, (b) $n$=7.

maximum current $J$=0.5 can only be reached at $\alpha=\beta=1$ [13–15].

Next we consider an $n$-lane ASEP with narrow entrances. Let us assume the effective entrance rates are given by $\alpha_l$. Therefore

$$\alpha_l = \alpha(1 - p_{L,l-1}) \quad \text{for } l > 1, \tag{1}$$

$$\alpha_1 = \alpha(1 - p_{L,n}). \tag{2}$$

Here $p_{L,l}$ denotes the density of site $L$ in lane $l$.

We first consider the LD phase in which all lanes are in equal low densities. The symmetric LD phase exists if

$$\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha_{eff} < \beta. \tag{3}$$

In the symmetric phase, the densities of site $L$ in all lanes are equal to each other:

$$p_{L,1} = p_{L,2} = \cdots = p_{L,n} = p_L. \tag{4}$$

Since all lanes are in LD, the flux in the lanes is

$$J = \frac{\alpha_{eff}}{\alpha_{eff} + 1}. \tag{5}$$

The flux can also be calculated by

$$J = p_L \beta. \tag{6}$$

Therefore, we have

$$p_L = \frac{\alpha_{eff}}{(\alpha_{eff} + 1)\beta}. \tag{7}$$

As a result,

$$\alpha_{eff} = \alpha(1 - p_L) = \alpha\left(1 - \frac{\alpha_{eff}}{(\alpha_{eff} + 1)\beta}\right). \tag{8}$$

This leads to

$$\beta\alpha_{eff}^2 + (\alpha + \beta - \alpha\beta)\alpha_{eff} - \alpha\beta = 0. \tag{9}$$

$\alpha_{eff}$ can be solved,

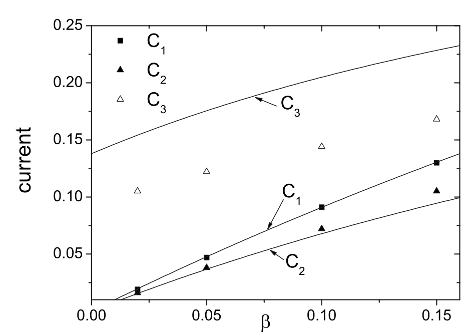


FIG. 17. Simulation results (scattered data) and analytical results (solid lines) of $C_1$, $C_2$, and $C_3$. The parameters are the same as in Fig. 13.

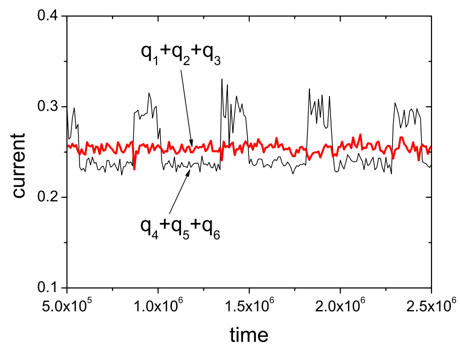


FIG. 18. (Color online) Time evolution of $q_1+q_2+q_3$ and $q_4+q_5+q_6$. The parameters are the same as in Fig. 13.

$$\alpha_{eff}=\frac{\alpha\beta-\alpha-\beta+\sqrt{(\alpha+\beta-\alpha\beta)^2+4\alpha\beta^2}}{2\beta}. \quad (10)$$

Together with the requirement $\alpha_{eff}<\beta$, we can obtain

$$\alpha<\beta+1, \quad (11)$$

which is always met. Note that Eq. (11) is only a necessary condition for the existence of the symmetric LD phase instead of a sufficient condition. Therefore, the phase boundary between the symmetric LD phase and other phases should be determined by other conditions.

Next we show it is not possible to have high densities in two successive lanes. Suppose lane 1 is in high density; we have $p_{L,1}=\frac{1}{\beta+1}$. Thus, $\alpha_2=\alpha(1-p_{L,1})=\alpha\left(1-\frac{1}{\beta+1}\right)=\frac{\alpha\beta}{\beta+1}<\beta$. Therefore, lane 2 cannot be in high density.

It is also not possible to have a high density in one lane and low densities in all other lanes. To demonstrate this, we first consider the case $n=3$. Suppose lane 1 is in high density; we have $p_{L,1}=\frac{1}{\beta+1}$ and $\alpha_2=\frac{\alpha\beta}{\beta+1}$. The flux in lane 2 is

$$J_2=\frac{\alpha_2}{\alpha_2+1}=\frac{\frac{\alpha\beta}{\beta+1}}{\frac{\alpha\beta}{\beta+1}+1}=\frac{\alpha\beta}{\alpha\beta+\beta+1}. \quad (12)$$

Thus,

$$p_{L,2}=\frac{\alpha}{\alpha\beta+\beta+1} \quad (13)$$

and

$$\alpha_3=\alpha(1-p_{L,2})=\frac{\alpha(\alpha\beta+\beta+1-\alpha)}{\alpha\beta+\beta+1}. \quad (14)$$

Since lane 3 is in low density, the flux is

$$J_3=\frac{\alpha_3}{\alpha_3+1}=\frac{\frac{\alpha(\alpha\beta+\beta+1-\alpha)}{\alpha\beta+\beta+1}}{\frac{\alpha(\alpha\beta+\beta+1-\alpha)}{\alpha\beta+\beta+1}+1}=\frac{\alpha(\alpha\beta+\beta+1-\alpha)}{\alpha(\alpha\beta+\beta+1-\alpha)+\alpha\beta+\beta+1}. \quad (15)$$

Thus,

$$p_{L,3}=\frac{\alpha(\alpha\beta+\beta+1-\alpha)}{\alpha\beta(\alpha\beta+\beta+1-\alpha)+(\alpha\beta+\beta+1)\beta} \quad (16)$$

and

$$\alpha_1=\alpha(1-p_{L,3})=\frac{\alpha^2\beta(\alpha\beta+\beta+1-\alpha)+(\alpha\beta+\beta+1)\alpha\beta-\alpha^2(\alpha\beta+\beta+1-\alpha)}{\alpha\beta(\alpha\beta+\beta+1-\alpha)+(\alpha\beta+\beta+1)\beta}. \quad (17)$$

Lane 1 in high density requires $\alpha_1>\beta$—i.e.,

$$\frac{\alpha^2\beta(\alpha\beta+\beta+1-\alpha)+(\alpha\beta+\beta+1)\alpha\beta-\alpha^2(\alpha\beta+\beta+1-\alpha)}{\alpha\beta(\alpha\beta+\beta+1-\alpha)+(\alpha\beta+\beta+1)\beta}>\beta. \quad (18)$$

Numerical investigation indicates that Eq. (18) is never met. As a result, lane 1 cannot be in high density. The analysis can be extended to $n>3$ straightforwardly, and the same conclusion is obtained.

For even $n$, the mean-field analysis for the HD-LD phase is the same as that in Ref. [12]: for the HD-LD phase to exist, the parameters should satisfy

$$\frac{\beta}{1-\beta}<\alpha\leqslant 1. \quad (19)$$

Similarly, on the line $\alpha=\frac{\beta}{1-\beta}$, the asymmetric LD-LD phase can exist. Also note that this is only a necessary condition for the existence of the asymmetric LD-LD phase instead of a sufficient condition.

For odd $n$, the period $T$ can be obtained from the mean-field approximation. Suppose lane $l$ begins to transit from low density to high density at time $t=t_0$. At this moment lane $l+1$ is in high density. The outflow rate is $J_{l+1,out}=\frac{\beta}{1+\beta}$, and the corresponding density is $\sigma_1=\frac{1}{\beta+1}$. The inflow rate into lane $l+1$ is $J_{l+1,in}=\frac{\alpha\beta}{\alpha\beta+\beta+1}$, and the corresponding density is $\sigma_2=\frac{\alpha\beta}{\alpha\beta+\beta+1}$ from Eq. (12). The velocity of the domain wall,

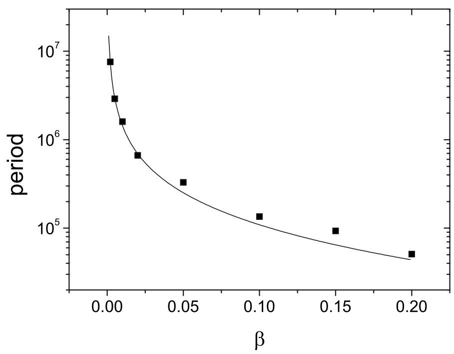


FIG. 19. Period of PS structure. The scattered point is from simulations, and the solid line is the analytical result. Parameters values: $L=1000$, $n=3$, $\alpha=0.8$.

$$v_{dw}=\frac{J_{l+1,out}-J_{l+1,in}}{\sigma_1-\sigma_2}=\frac{\frac{\beta}{1+\beta}-\frac{\alpha\beta}{\alpha\beta+\beta+1}}{\frac{1}{\beta+1}-\frac{\alpha\beta}{\alpha\beta+\beta+1}}=\frac{\beta^2+\beta-\alpha\beta}{\beta+1-\alpha\beta^2}. \quad (20)$$

Therefore,

$$T_d=\frac{L}{v_{dw}}=\frac{L(\beta+1-\alpha\beta^2)}{\beta^2+\beta-\alpha\beta}$$

and the period $T$ is

$$T=nT_d=\frac{nL(\beta+1-\alpha\beta^2)}{\beta^2+\beta-\alpha\beta}. \quad (21)$$

Our analytical results are in good agreement with the simulation results when $\beta$ is small (Fig. 19). Nevertheless, with an increase of $\beta$, the analytical results deviate from simulations, due to the correlation being neglected in the mean-field approximation. When $\beta$ is beyond the phase boundary, the PS phase disappears and the mean-field results (20) and (21) become invalid.

Finally, when $\alpha=\beta=1$, the maximum current can be achieved if $L$ is odd. If $L$ is even, the current depends on the system size as $J=\frac{L/2}{L+1}$. Therefore, the maximum current can be achieved only when $L\rightarrow\infty$. This is the same as in the case of $n=2$.

The phase diagram predicted by mean-field calculations for even $n$ is also shown in Fig. 2. As indicated in Ref. [12], the phase structure deviates from Monte Carlo simulations because correlations are important in the dynamics of this system and they are neglected in the mean-field approximation.

## IV. CONCLUSION

In this paper, we have studied an $n$-lane totally asymmetric simple exclusion process (TASEP) with narrow entrances under parallel update. It is shown that the results depend on the value of $n$. If $n$ is an even number, the results are essentially the same as $n=2$: two symmetry-breaking phases—i.e., the HD-LD phase and LD-LD phase—are identified, although the latter may disappear in the thermodynamic limit. In contrast, if $n$ is an odd number, a periodic structure is observed and the period is found to be proportional to lane number $n$ and system size $L$. Furthermore, when $\alpha=1$, the seesaw phase observed in the case of $n=2$ disappears and the HD-LD phase or symmetric LD phase appears.

Some mean-field calculations are also presented. It is shown that it is not possible to have high densities in two successive lanes and also not possible to have high density in one lane and low densities in all other lanes. For even $n$, the mean-field analysis for the HD-LD phase and LD-LD phase is the same as in the case of $n=2$. For odd $n$, we have obtained the period $T$ and it is in good agreement with simulations for small $\beta$ but deviates from simulation results for large $\beta$ because correlation is neglected in the mean-field approximation.

We also would like to mention that for finite $L$ and arbitrary large $n$, the results will be different and the model dynamics will not be sensible to whether the number of lanes is even or odd, because an increasing number of stochastic bonds along with $n$ increases fluctuations and ruins the coherence, which leads to a smoothing out the differences between the two cases. Under this circumstance, neither spontaneous symmetry breaking nor periodic structure could be observed.

In our future work, we will study the $n$-lane system considering stochastic hopping in the bulk—i.e., $q<1$. Furthermore, the $n$-lane system with random sequential update rules is also needed to be investigated and compared with that reported in this paper.

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[16] One problem is, if $\tau \propto \exp[b(L)n^{\gamma(L)}]$ holds for general $n$ and $L$, we have $\ln(\tau) \propto b(L)n^{\gamma(L)}$. This contradicts with the results that $\tau$ grows exponentially with the system size $L$. One possibility is $\gamma(L) \to \text{const}$ with an increase of $L$. Further investigations are needed for verification.